A GOAL PROGRAMMING DODEL WITH STOCHASTIC GOAL CONSTRAINTS

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ABSTRACT

This paper presents a goal-programming model in which the parameters are stochastic. The study extends the one-sided probabilistic goal constraints to the case of two-sided probabilistic goal constraints with proper formulation, which preserves the original characteristic of goal programming. This new two-sided probabilistic goal-constrained method is more efficient than the existing methods in that it has fewer constraints and decision variables.

The resulting chance-constrained goal-programming method provides an effective way of adapting the simplex method, which takes into account the nonlinear behavior of the parameters of a model. Furthermore, this proposed method is illustrated through a financial and production example.

Key words: goal, modeling, probability, programming, multi-objective, and stochastic.

1. INTRODUCTION

Goal programming, introduced by Charnes and Cooper (1), deals with the problem of achieving a set of conflicting goals. The objective function searches to minimize deviations from the set of pre-assigned goals. Two types of goals are commonly used in goal programming models. The first type is a two-sided goal applicable to goals, which must be achieved exactly; any deviation, either upward or downward, would result in penalty. The second type is a one-sided goal with which only upward or downward deviation would be penalized. When uncertainties exist in the goal-programming problem, the chance-constrained formulation is often used (2).

There are many applications of chance-constrained goal programming to various areas including capital management (3), bank liquidity management (4), capital budgeting (5), response resources for marine pollution disasters (6), production planning (7), mental health planning (8), and others. The great majority of chance-constrained goal programming models consider one-sided goals with very few having two-sided goals. The process most researchers used in deriving the associated deterministic equivalent for the goals, in the author's opinion, is not entirely correct in two aspects: 1) violation of basic probability law, and 2) alteration of original goal. These points are elaborated in the next section. The purpose of this paper is to propose an alternative formulation for a chance-constrained goal-programming model which preserves the original characteristics of the problem without changing the problem's goal. The formulation utilizes the concept similar to the confidence interval in statistics. This is explained in the following.

2. CURRENT FORMULATION OF CHANCE-CONSTRAINED GOAL PROGRAMMING

Consider a one-sided goal to which the over-achievement would result in an undesirable penalty. That is, the objective is to determine decision variables $\mathbf{x} = (x_1, x_2, \ldots, x_n)^t$ which makes, if possible, the objective function value not exceed the pre-specified goal, *G*, as

$$\sum_{j=1}^{n} c_j x_j \le G \qquad (1)$$

in which $\mathbf{c} = (c_1, c_2, \ldots c_n)$ is a vector of objective function coefficients. The deterministic goal programming model for such goal is expressed as

$$\sum_{j=1}^{n} c_j x_j + D^- - D^+ = G \qquad (2)$$

with D^- and D^+, respectively, representing the under- and over-achievement with respect to the goal in which positive penalty is associated with D^+ in the objective function. It should be noted that throughout this paper a vector is shown by bold lowercase letters, and a matrix is shown by bold uppercase letters. When the elements in ***c*** are random variables, two approaches are found in the literature that convert the deterministic equation into the chance-constrained one. One approach is to utilize Equation 2 by simply converting it into a probabilistic statement as

$$\mathrm{P}\left[\sum_{j=1}^{n} c_j x_j + D^- - D^+ = G\right] \ge \boldsymbol{a} \qquad (3)$$

with a being the goal compliance reliability. The error committed in this formulation is that the modelers failed to recognize that the probability of a continuous random variable equaling to a fixed value is zero. This type of error is found, especially, in problems with two-sided goals.

The other incorrect approach commonly used to derive the chance-constrained goal programming (CCGP) formulation (7, 8) starts with deriving the chance constraint from Equation 1 as

$$\mathrm{P}\left[\sum_{j=1}^{n} c_j x_j \leq G\right] \geq a \tag{4}$$

The corresponding deterministic equivalent of Equation 4 is derived as

$$\sum_{j=1}^{n} E(c_j) x_j + Z_a [\mathbf{x}'\Sigma\mathbf{x}]^{.5} \leq G \tag{5}$$

in which $E(\mathbf{c}_j)$ is the mean value of the *j-th* objective coefficient and Σ is the variance-covariance matrix of the random objective coefficients. The chance-constrained goal programming equation is then obtained by adding deviational variables in Equation 5 as

$$\sum_{j=1}^{n} E(\mathbf{c}_j)\mathbf{x}_j + D^- - D^+ = G - Z_a [\mathbf{x}'\Sigma\mathbf{x}]^{.5}. \tag{6}$$

Comparing Equations 2 and 6, it is observed that the original goal G in Equation 2 has been changed in the chance-constrained formation which is not correct. Similarly, the above argument is valid for the other type of one-sided goal in which under-achievement is to be penalized.

3. PROPOSED FORMULATION OF The CCGP MODEL

The concept of the proposed chance-constrained goal formulation is considered for the three possible goal types of stochastic goal programming. The first type represents a two-sided goal for which the upward and downward deviations are to be penalized. In this case, it is desired to obtain a shortest interval that attains the goal, a pre-assigned reliability of a_r. Second (upward deviation) and third (downward deviation) types represent one-sided goals. In these cases, only upward or downward deviations are to be penalized. The reliability of attaining each goal is denoted by a_s and a_t, respectively.

A general form of the chance-constrained goal -programming model, based on the goal programming format, may be formulated as

$$\text{Minimize } D_0 = \sum_{r=1}^{R}(D_r^+ + D_r^-) + \sum_{s=1}^{S} D_s^+ + \sum_{t=1}^{T} D_t^- \tag{7}$$

Subject to

$$P\left[G_r - D_r^- \le \mathbf{c}_r^{'}\mathbf{x} \le G_r + D_r^+\right] > a_r, \qquad r = 1,...,R \tag{8}$$

$$P\left[\mathbf{c}_s^{'}\mathbf{x} \le G_s + D_s^+\right] > a_s, \qquad s = 1,2,...,S \tag{9}$$

$$P\left[\mathbf{c}_t^{'}\mathbf{x} \ge G_t - D_t^-\right] > a_t, \qquad t = 1,2\ldots,T \tag{10}$$

$$P\left[A\mathbf{x} \le \mathbf{b}\right] \ge b \tag{11}$$

$$x \geq 0 \tag{12}$$

$$D_r^+, D_r^-, D_s^+, D_t^- \geq 0, \qquad \text{for all } r, s, \text{ and } t. \tag{13}$$

Where R is the number of two-sided goals, S is the number of one-sided goals for upside deviation, and T is the number of one-sided goals for downside deviation. In addition, a_r, a_s and a_t are pre-assigned reliabilities of attaining the corresponding goals.

Constraint 11 represents the regular constraint in a probabilistic form which takes into account the randomness of $\mathbf{A}$ and $\mathbf{b}$. It can be represented as a deterministic constraint when the elements of $\mathbf{A}$ and $\mathbf{b}$ are all constants.

Constraints 8 through 11 are all probabilistic in a general setting. In order to solve a stochastic goal-programming problem, these constraints must be transformed to their respective deterministic equivalents. These transformations will be presented in the next section.

3.1. Derivation of Deterministic Equivalents for Chance-Constrained Goals

3.1.1. One-sided goals:

The deterministic equivalent for a one-sided goal with only upward deviation can be derived as

$$E(\mathbf{c}_s^{'})\mathbf{x} - D_x^+ + \left[F_z^{-1}(a_s)\right] \bullet \left[\mathbf{x}'\Sigma_\mathbf{s}\mathbf{x}\right]^{.5} \leq G_s \tag{14}$$

and with downward deviation as

$$E(\mathbf{c}_t^{'})\mathbf{x} - D_s^- + \left[F_z^{-1}(1 - a_t)\right] \bullet \left[\mathbf{x}'\Sigma_\mathbf{t}\mathbf{x}\right]^{.5} \geq G_t \qquad (15)$$

where F_z^{-1} is the inverse CDF of the standardized random variable **c'x**. In case that **c'x** is a normal random variable, then F is a standard normal CDF. These results of deterministic equivalents for one-sided goal can be found elsewhere (9).

3.1.2. Two-sided goals:

Consider a two-sided goal constraint:

$$P\left[G_r - D_r^- \leq \mathbf{c}_r^{'}\mathbf{x} \leq G_r + D_r^+\right] > a_r \qquad (16)$$

It can be expressed as

$$P\left[\mathbf{c}_r^{'}\mathbf{x} \leq G_r + D_r^+\right] - P\left[\mathbf{c}_r^{'}\mathbf{x} \leq G_r - D_r^-\right] > a_r \quad . \qquad (17)$$

Assume that the elements in $\mathbf{c_r}$ are independent normal random variables with mean $E(c_j)$ and variance $\operatorname{var}(c_j)$. Standardizing Equation 17 leads to

$$\Phi\left[\frac{G_r + D_r^+ - E(\mathbf{c}_r^{'})\mathbf{x}}{\left[\mathbf{x}'\Sigma_\mathbf{r}\mathbf{x}\right]^{.5}}\right] - \Phi\left[\frac{G_r - D_r^- - E(\mathbf{c}_\mathbf{r}^{'})\mathbf{x}}{\left[\mathbf{x}'\Sigma_\mathbf{r}\mathbf{x}\right]^{.5}}\right] > a_r \qquad (18)$$

where Φ is the CDF of the standard normal random variable.

For the purpose of simplifying the presentation regarding Equation 18, let the subscripts be dropped and the first and second function be denoted as Φ^+ and Φ^-, respectively. A direct method to obtain a deterministic equivalent of Equation 18 is to replace constraint equation 18 by the following three constraints.

$$\Phi^{+}\left[\frac{G+D^{+}-E(\mathbf{c}')\mathbf{x}}{[\mathbf{x}'\Sigma\mathbf{x}]^{.5}}\right]=a' \tag{19}$$

$$\Phi^{-}\left[\frac{G-D^{-}-E(\mathbf{c}')\mathbf{x}}{[\mathbf{x}'\Sigma\mathbf{x}]^{.5}}\right]=1-a'' \tag{20}$$

$$a'-a''>a \tag{21}$$

Here a' and a'' are the unknown reliabilities for achieving each new equation. In doing so, two equations (one linear and one nonlinear) and decision variables (a' and a'') are added to this model. Hence, in a problem having several two-sided goals, this approach could result in an additional large number of decision variables and nonlinear constraints. In the following, a derivation is presented that leads to a formulation having only one extra linear constraint with no new decision variable added to the model.

As argued previously, the objective of a two-sided goal is to find the solution having the shortest interval $(G-D^{-}, G+D^{+})$ with a goal reliability compliance. Therefore, derivation of the deterministic equivalent of a two-sided goal can be written as

$$Min \quad D_0 = D^{+} + D^{-}. \tag{22}$$

subject to

$$\Phi^{+}\left[\frac{G+D^{+}-E(\mathbf{c}')\mathbf{x}}{[\mathbf{x}'\Sigma\mathbf{x}]^{.5}}\right]-\Phi^{-}\left[\frac{G-D^{-}-E(\mathbf{c}')\mathbf{x}}{[\mathbf{x}'\Sigma\mathbf{x}]^{.5}}\right]>a \tag{23}$$

To solve the mathematical programming problem 22 - 23, the Lagrangian method is used. The Lagrangian function is thus given by

$$Min\ \ L(D^{+}, D^{-}, l) = D^{+} + D^{-} + l\,(\Phi^{+} - \Phi^{-} - a) \tag{24}$$

in which l is the Lagrangian multiplier. The solution to the above Lagrangian function $L(D^{+}, D^{-}, l)$ must satisfy

$$dL \big/ l\,D^{+} = 1 + l \left\{ f^{+} \big/ [\mathbf{x}'\Sigma\mathbf{x}]^{.5} \right\} = 0 \tag{25}$$

$$dL \big/ d\,D^{-} = 1 + l \left\{ f^{-} \big/ [\mathbf{x}'\Sigma\mathbf{x}]^{.5} \right\} = 0 \tag{26}$$

$$dL \big/ d\,l = \Phi^{+} - \Phi^{-} - a = 0 \tag{27}$$

in which f^{+} and f^{-} are the pdf of the standard normal random variables. Solving Equations 25 and 26 simultaneously, the following results were obtained:

$$l\,(\Phi^{+} - \Phi^{-}) \big/ [\mathbf{x}'\Sigma\mathbf{x}]^{.5} = 0 \tag{28}$$

Since the square root of $\mathbf{x}'\Sigma\mathbf{x}$ is a positive value and l is a non-zero, thus

$$f^{+} - f^{-} = 0 \tag{29}$$

which implies two possible solutions to the problem:

$$\text{Either}\ \ \frac{G + D^{+} - E(\mathbf{c}')\mathbf{x}}{[\mathbf{x}'\Sigma\mathbf{x}]^{.5}} = \frac{G - D^{-} - E(\mathbf{c}')\mathbf{x}}{[\mathbf{x}'\Sigma\mathbf{x}]^{.5}} \tag{30}$$

$$\text{or } \frac{G + D^{+} - E(\mathbf{c}')\mathbf{x}}{\left[\mathbf{x}'\Sigma\mathbf{x}\right]^{.5}} = -\frac{G - D^{-} - E(\mathbf{c}')\mathbf{x}}{\left[\mathbf{x}'\Sigma\mathbf{x}\right]^{.5}} \tag{31}$$

Equation 30 indicates that $D^{+} = D^{-}$. By substituting the equated value of D^{+} in the original probability statement, Equation 16, yields

$$P\left[G - D^{-} \le \mathbf{c}'\mathbf{x} \le G - D^{-}\right] > a \tag{32}$$

which is equal to

$$P\left[\mathbf{c}'\mathbf{x} = G - D^{-}\right] > a\ . \tag{33}$$

Since the model deals with a continuous random variable and G and D^{-} are non-random, then the probability of a random variable to be equal to a fixed value is equal to zero. Hence, the solution given by Equation 30 is infeasible.

The solution represented by Equation 31 is equivalent to

$$G + D^{+} - E(\mathbf{c}')\mathbf{x} = -G + D^{-} + E(\mathbf{c}')\mathbf{x} \tag{34}$$

which can be written as

$$2E(\mathbf{c}')\mathbf{x} - D^{+} + D^{-} = 2G. \tag{35}$$

Substituting the results obtained from Equation 31 into Equation 23, we have

$$\Phi\left[\frac{G + D^{+} - E(\mathbf{c}')\mathbf{x}}{\left[\mathbf{x}'\Sigma\mathbf{x}\right]^{.5}}\right] - \Phi\left[-\frac{G + D^{+} - E(\mathbf{c}')\mathbf{x}}{\left[\mathbf{x}'\Sigma\mathbf{x}\right]^{.5}}\right] > a\,. \tag{36}$$

After some algebraic manipulations on Equation 36, the following equation can be obtained:

$$\Phi\left[\frac{G + D^{+} - E(\mathbf{c}')\mathbf{x}}{\left[\mathbf{x}'\Sigma\mathbf{x}\right]^{.5}}\right] > \frac{(1+a)}{2}. \tag{37}$$

The deterministic equivalent of Equation 37 then is

$$\frac{G + D^{+} - E(\mathbf{c}')\mathbf{x}}{\left[\mathbf{x}'\Sigma\mathbf{x}\right]^{.5}} > \Phi_z^{-1}\left[(1+a \big/ 2\right] \tag{38}$$

where Φ_z^{-1} is the inverse CDF of the standard normal random variable.

3.2. Summary of Proposed CCGP Model

The derivations of deterministic equivalents of chance-constrained two-sided and one-sided goals have been given in the previous section. A general chance-constrained goal-programming model should include all of these constraints in order to take into consideration any possible conflicting two-sided and one-sided goals. Thus, the general model, based on Equations 7, 14, 15, 35, and 38 may be summarized as

$$Min\ D_0 = \sum_{r=1}^{R}(D_r^{+} + D_r^{-}) + \sum_{s=1}^{S} D_s^{+} + \sum_{t=1}^{T} D_t^{+} \tag{39}$$

subject to

$$E(\mathbf{c}_r^{'})\mathbf{x} - .5D_r^{+} + .5D_r^{-} = G_r,\ \text{for all } r \tag{40}$$

$$E(\mathbf{c}_r^{'})\mathbf{x} - D_r^{+} + \Phi_z^{-1}\left[(1-a_r)/2\right] \bullet \left[\mathbf{x}'\Sigma_r\mathbf{x}\right]^{.5} \leq G_r,\ \text{for all } r \tag{41}$$

$$E(\mathbf{c}_S^{'})\mathbf{x} - D_s^{+} + \Phi_Z^{-1}\left[a_s\right] \bullet \left[\mathbf{x}'\Sigma_s\mathbf{x}\right]^{.5} \leq G_s,\ \text{for all } s \tag{42}$$

$$E(\mathbf{c}_t^{'})\mathbf{x} + D_t^{-} + \Phi_z^{-1}\left[1-a_t\right] \bullet \left[\mathbf{x}'\Sigma_t\mathbf{x}\right]^{.5} \geq G_t,\ \text{for all } t \tag{43}$$

$$\mathbf{x} \geq \mathbf{0} \tag{44}$$

$$D_r^+, D_r^-, D_s^+, D_t^- \geq 0 \text{, for all } r, s, \text{ and } t. \tag{45}$$

The above model can be generalized even further by including the regular constraints of a deterministic form, a probabilistic form, or a mixture of both in the model.

4. A PROPOSED SOLUTION ALGORITHM

A deterministic goal programming model follows a linear programming format which can be easily solved by the simplex algorithm. However, the deterministic equivalent transformation of chance-constrained goal programming constraints leads to the presence of non-linearity, which cannot be solved directly by the linear programming technique. Therefore, the problem becomes one of non-linear optimization, which can be solved by various non-linear programming techniques such as the generalized reduced gradient technique (10).

Alternatively, this paper adopts a procedure to linearize the non-linear terms of the chance constraints in the CCGP model similar to the successive linear programming procedure (11) and solve the linearized model iteratively. The "linearized" constraints in the CCGP model are obtained by moving the non-linear terms to the RHS of the constraints and can be written as

$$E(\mathbf{c}_r^{'})\mathbf{x} - D_r^+ \leq G_r - \Phi_z^{-1}\left[{(1+a_r}\big/{2}\right] \bullet \left[\mathbf{x}'^{\,0}\Sigma_r\mathbf{x}\right]^{.5} \tag{46}$$

$$E(\mathbf{c}_s^{'})\mathbf{x} - D_s^{+} \leq G_s - \Phi_z^{-1}[a_s] \bullet \left[\mathbf{x}'^{0}\Sigma_s\mathbf{x}\right]^{.5} \qquad (47)$$

$$E(\mathbf{c}_t^{'})\mathbf{x} + D_t^{-} \geq G_t - \Phi_z^{-1}[1 - a_t] \bullet \left[\mathbf{x}'^{0}\Sigma_t\mathbf{x}\right]^{.5} \qquad (48)$$

where $\mathbf{x}^0$ is an assumed solution vector to the optimal CCGP model. Consequently, the linearlized CCGP model can be solved by the LP technique iteratively, each time comparing the values of the current solutions with those obtained in the previous iteration. Then, updating the assumed solution values, using them to compute the variance term on the right hand sides, until convergence criteria are met. Of course, alternative stopping rules, such as specifying the maximum number of iterations, can also be added in order to prevent excessive iteration during the computation. However, prior to the application of these procedures, an assumption for the distribution of the standardized random variable Z must be made so that the terms $\Phi_z^{-1}\left[(1+a_r)/2\right]$, $\Phi_z^{-1}[a_s]$, and $\Phi_z^{-1}[1-a_t]$ in Equations 46 through 48 can be evaluated.

Due to the non-linear nature of the CCGP model, the optimum solution obtained, in general, cannot be guaranteed to be the global optimum. Thus, it is suggested that a few runs of these procedures with different initial solutions should be carried out to ensure that the model solution converges to an overall optimum.

5. ILLUSTRATION

A financial and production example illustrating the proposed model in this paper was adopted from Weingartner (12), and Hawkins and Adams (13). The problem considers nine mutually exclusive projects with given net present values for each project and a certain configuration of fund outlay over a two-year period. The objective is to maximize the present values of these investments in the context of a fraction of the adopted investment proposal, given a budget constraint of $50 for the first period and $20 for the second period. Hawkins and Adams (13) modified the problem under the assumption that this is a manufacturing firm in which each investment proposal is expected to yield a certain amount of revenue in each period and utilize a specified number of man-hours per day. Table 1 lists the investment proposals along with their net present values and the present values of outlays, sales, and man-hours for each period.

Assume that the top management of the firm establishes the following goals:

a) The project as a whole must yield a net present value of at least $32.40. So it is a one-sided goal because there is no penalty if it is more than this amount.

b) The sales must be at least $70 for the first period. It again is a one-sided goal because if it is more than this amount, it is desirable but if it is less, it causes carrying costs or the liquidation of products at a much lower price.

c) The sales must be at least $84 for the second period. This is a one-sided goal similar to goal b).

d) The man-hours of labor per day must be exactly 40 for the first period. This is a two-sided goal because any deviations (whether upward such as payment for overtime or downward such as idle time of employees) are not desirable.

e) The man-hours of labor per day are to be exactly 40 for the second period. Again, it is a two-sided goal similar to goal d).

The problem has so far dealt with the certainty case. However, because of uncertainty associated with the objective function coefficients in each goal, the top management wishes to achieve the goals with certain pre-assigned reliabilities $(a_1, a_2, \ldots a_k)$. In addition, it is assumed that the objective function coefficients are independent normal random variables, with the means given in Table 1 and standard deviations equal to some percentage of the means.

The CCGP model for this problem can be formulated as

$$Min\ D_0 = D_1^- + D_2^- + D_3^- + D_4^- + D_5^- + D_5^+ \tag{49}$$

subject to

$$P\left[\mathbf{c}_1^{'}\mathbf{x} \geq G_1 - D_1^-\right] > a_1 \tag{50}$$

$$P\left[\mathbf{c}_2^{'}\mathbf{x} \geq G_2 - D_2^-\right] > a_2 \tag{51}$$

$$P\left[\mathbf{c}_3^{'}\mathbf{x} \geq G_3 - D_3^-\right] > a_3 \tag{52}$$

$$P\left[G_4 - D_4^- \leq \mathbf{c}_4^{'}\mathbf{x} \leq G_4 + D_4^+\right] \geq a_4 \tag{53}$$

$$P\left[G_5 - D_5^- \leq \mathbf{c}_5^{'}\mathbf{x} \leq G_5 + D_5^+\right] \geq a_5 \tag{54}$$

$$\mathbf{a}_1^{'}\mathbf{x} \leq b_1 \tag{55}$$

$$\mathbf{a}_2^{'}\mathbf{x} \leq b_2 \tag{56}$$

$$\mathbf{1} \geq \mathbf{x} \geq \mathbf{0} \tag{57}$$

$$D_1^-, D_2^-, D_3^-, D_4^-, D_4^+, D_5^-, D_5^+ \geq 0 \tag{58}$$

where vector $\mathbf{c}_1$ through $\mathbf{c}_5$ the coefficients of goal constraints, represent the present values of investment, the sales in period 1 and period 2, and man-hours in period 1 and period 2, respectively. Vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ in constraints 55 and 56 are the technological coefficients of regular deterministic constraints representing the present values of outlay in period 1 and period 2, respectively.

To examine the effects of the specified reliability levels and the level of uncertainty for the optimal solutions, the financial and production example with 5%, 10%, 25%, and 50% standard deviations and various reliabilities as 85%, 90%, and 95% were solved and the results are given in Table 2.

In examining the results presented in Table 2, the total amount of optimal deviations is sensitive to various standard deviations and reliabilities. For a given uncertainty level of model parameters, the total amount of optimal deviations is increased as the reliability requirement of the goal constraints increases. For instance, at 5% standard deviations for the coefficients in goal constraints, an increase in reliability from .85 to .95 for all goals results in an increase in the total amount of optimal deviations from 32.6 to 35.0. This indicates that in order to achieve the goals with high reliability, more total deviations should be anticipated in the objective functions. By increasing the standard deviations for the same level of reliability, as might be expected, the value of the objective function is increased. For example, by increasing the standard deviations for all coefficients of goal constraints from 5% to 25% under a 90% reliability for each goal, the optimal value of the objective function is increased from 33.6 to 54.0. Computationally, it appears that the number of iterations required to converge also increases when the uncertainty of model parameters increases.

The tabulated values in Table 2 present a good indication of relationships and how sensitive the total amount of optimal deviations is to various standard deviations, reliabilities, and the number of iterations. These results can be explained by the fact that, as the reliability for each goal constraint is increased, it is equivalent to imposing stricter standard deviations on random coefficients in order to minimize the total deviations.

6. CONCLUSION

The CCGP model has been used in various areas of multi-objective problems. This research has proposed a new formulation for chance-constrained goal programming model which preserves the original characteristic of the problem without changing the problem's goal. It also utilizes the concept of the confidence interval due to the randomness of objective coefficients which are continuous random variables.

The results obtained from applying the model to a multiple-objective example revealed the contribution of the model to the managerial decisions in terms of the reliability specified for the goal accomplishment and the total amount of goals deviations.

Finally, because of the uncertainty one may encounter in a real world problem, the developed CCGP model not only takes into account the risk and uncertainty involved in the coefficients of goal and regular constraints, but also has plausible representation of the nature of the goals, whether they are one-sided or two-sided.

Table 1: Data of the Investment Proposals Example Used in the CCGP Model

Investment Project	**PV* of Investment**	**PV of Outlay**		**Sales**		**Man-Hours**	
		Pd 1	**Pd 2**	**Pd 1**	**Pd 2**	**Pd 1**	**Pd 2**
1	$14	$12	$3	$14	$15	10	12
2	17	54	7	30	42	16	16
3	17	6	6	13	16	13	13
4	15	6	2	11	12	9	13
5	40	30	35	53	52	19	16
6	12	6	6	10	14	14	14
7	14	48	4	32	34	7	9
8	10	36	3	21	28	15	22
9	12	18	3	12	21	8	13

*PV = Present Value

Table 2: Results From the Execution of Example CCGP Problem With Various Standard Deviations and Reliabilities But Equal Weight Criterion

S.D.	**5%**			**10%**			**25%**			**50%**		
Reli	**.85**	**.90**	**.95**	**.85**	**.90**	**.95**	**.85**	**.90**	**.95**	**.85**	**.90**	**.95**
x_1	.332	.305	.266	.220	.168	.090	.000	.000	.000	000	.000	1.00
x_2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
x_3	1.00	1.00	1.00	1.00	1.00	1.00	.918	.816	.682	.536	.642	.327
x_4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
x_5	.251	.253	.255	.258	.261	.255	.285	.302	.325	.350	.332	.324
x_6	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
x_7	.551	.557	.564	.575	.586	.602	.623	.625	.627	.630	.628	.423
x_8	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
x_9	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
D_1^-	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
D_2^-	11.8	12.3	13.0	13.8	14.7	16.2	19.7	21.9	25.1	28.9	32.2	33.4
D_3^-	20.8	21.3	22.0	22.9	24.0	25.5	29.5	32.1	35.8	40.2	43.3	45.0
D_4^-	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
D_4^+	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	5.27
D_5^-	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

D_5^+	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	3.32	14.1
D_0	32.6	33.6	35.0	36.7	38.7	41.7	49.2	54.0	60.9	69.1	78.8	97.7
#Ite	3	3	3	3	3	3	3	4	4	4	4	4

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